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High Fidelity Modeling of Field-Reversed Configuration (FRC) Thrusters

Space Propulsion and Power Portfolio Review
28 September – 02 October 2015

Justin Koo
AFRL/RQRS

Overview

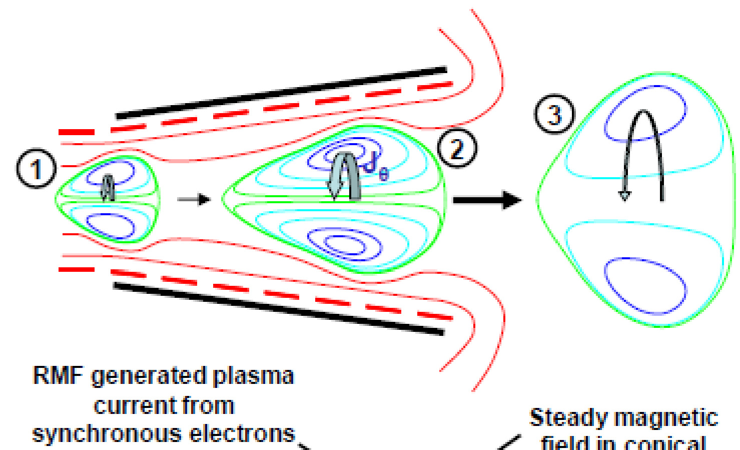
- **Background**
  - Branch-level strategy
  - Status of experimental efforts
- **Project Description**
  - Goals/Objectives/Technical Challenge/Approach
- **Progress Update**
  - Low fidelity (Hugrass) status
  - High fidelity (Multifluid) status
  - 2-fluid FRC simulation
- **Conclusions**

Background

FRC propulsion

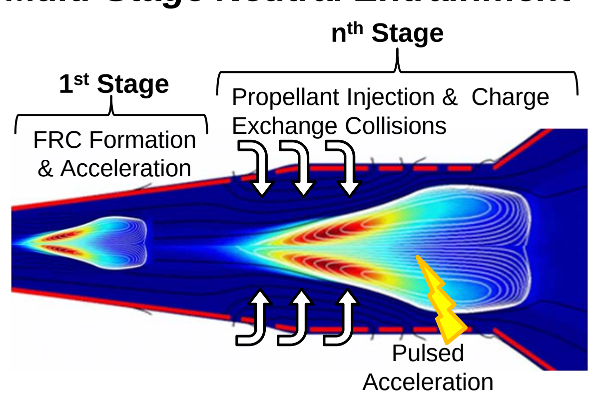
- **Field Reversed Configuration (FRC) is offshoot of fusion research**
  - Ionization by rotating B-field
  - Pulsed inductive jxB acceleration
  - Magnetically insulated, plasmoid accelerated downstream

- **Key attributes**
  - Very low mass (estimate ~1-2 kg/kW)
  - Efficiency comparable to or higher than Hall thrusters (predicted)
  - Operates on diverse propellants; potential for multi-mode application
  - Pulsed operation provides near-constant efficiency over wide power range



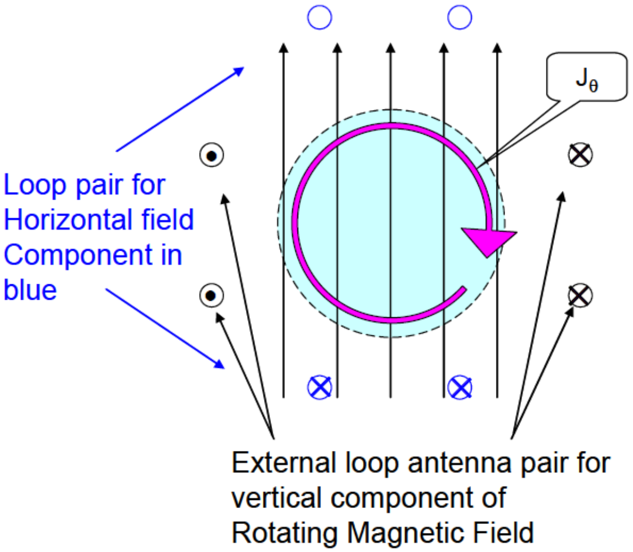
$$F_z = j_\theta \times B_r$$

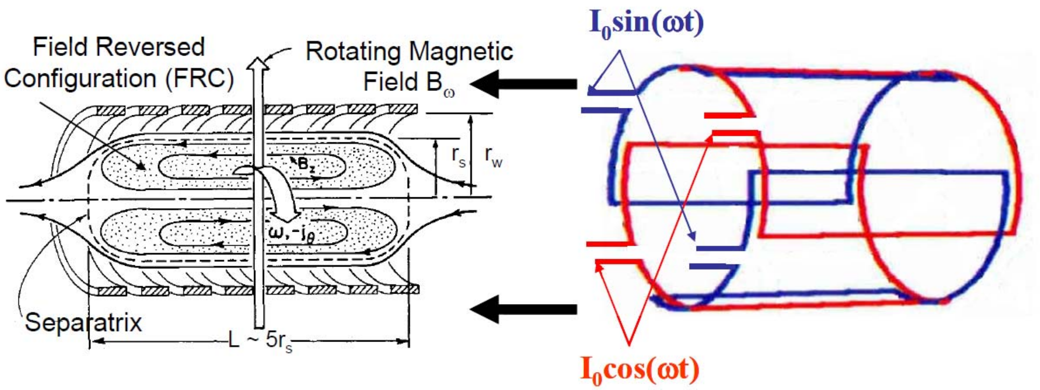
Multi-Stage Neutral Entrainment



Rotating Magnetic Field (RMF)

Electrodeless Lorentz Force (ELF) Rotating Magnetic Field (RMF) thruster





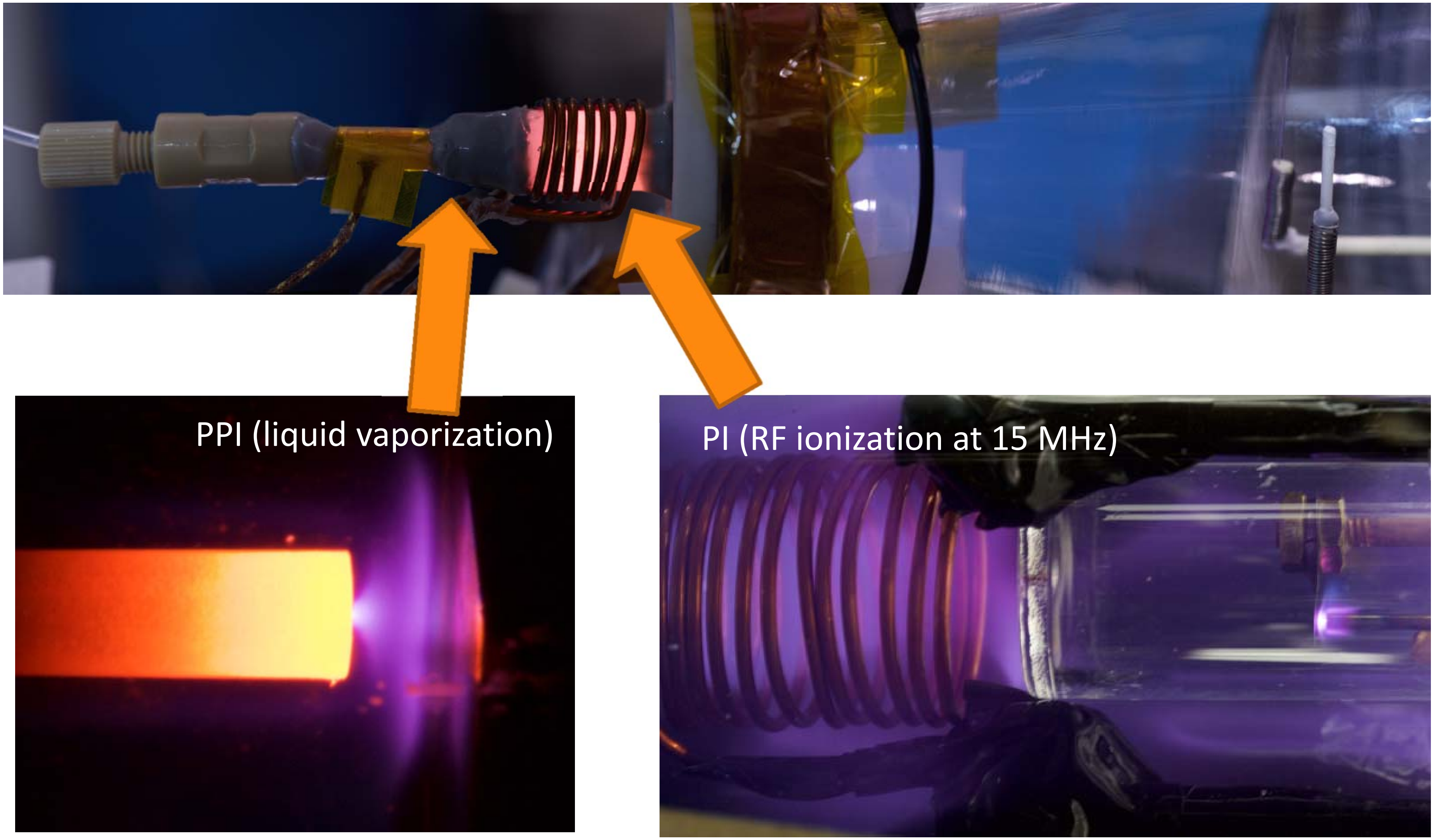
RMF FRC thruster design appears well suited for low power operation; different set of computational challenges than theta-pinch RMF simulation

Slough, J, Kirtley, D and Weber T, **Pulsed Plasmoid Propulsion: The ELF Thruster ,** IEPC-2009-265, *31th International Electric Propulsion Conference, Ann Arbor, Michigan, September 20-24, 2009.*

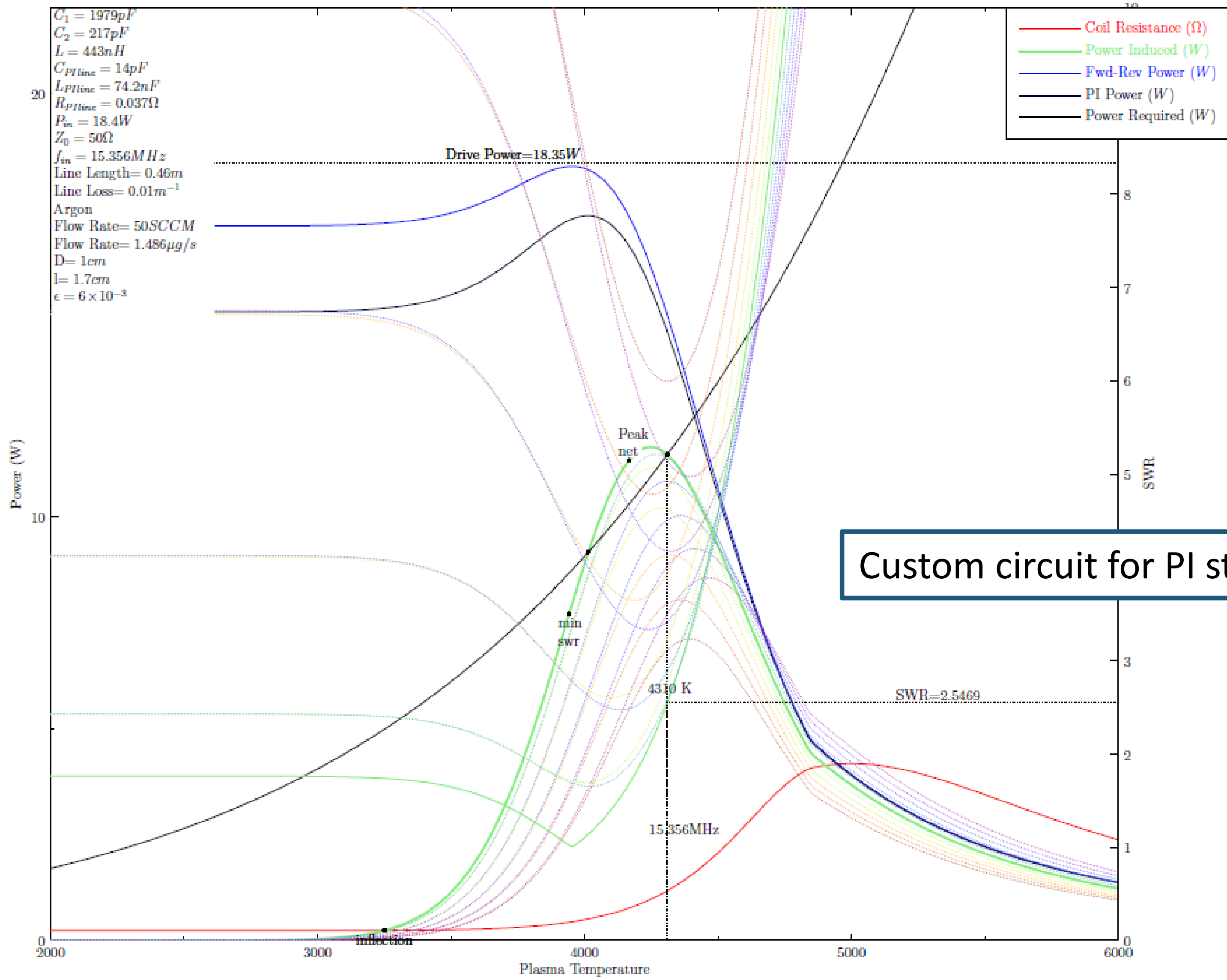
Branch-level FRC development strategy

- **In-house**
  - Computational 6.1 program to develop numerical tools (this program)
  - Experimental 6.2 program to generate validation data
    - Briefly attempted theta-pinch configuration, failed to make sufficient progress (limited amount of data)
    - Switched to rotating magnetic field (RMF) configuration
    - Simultaneous development of PI/PPI systems
- **External**
  - Leveraging SBIRs to develop engineering-level FRC models for testing
  - Testing thrusters at MSNW LLC (Redmond, WA) and AFRL/RQRS (Edwards AFB, CA)

Experimental efforts (1)



Experimental efforts (2)



Custom circuit for PI stage

Distribution A – Approved for public release; distribution unlimited

Project Description

Justification

- **Some computational models exist**
  - Neutral flow dynamics
  - Fully-ionized FRC translation
  - Collisional kinetic of charge-exchange
- **The right integration of numerical models does not exist**
  - Partially ionized magnetized plasmas are difficult to simulate
  - Modern diagnostics include internal/external probes and spectroscopy
  - Simulation of realistic FRCs requires specific physic modules such as coil-plasma interaction models

Fundamental question is how to scale FRCs to run efficiently at low power levels – difficult/impossible without simulation

Goals

- **Pre-ionization –** need to generate some amount of seed plasma; for multi-plasmoid operation, first pulse is the most difficult; possible to consider many different configurations (e.g. liquid phase propellants)
  - Very specific to configuration
  - Introduction of multiphase and chemistry greatly complicate physics
- **Formation –** go from seed plasma to strongly ionized closed-field plasmoid; kinetic effects expected in first stage; interaction between coil/plasma drive process
- **Translation –** plasma remain almost fully ionized; interested primarily in collisional effects as plasma impact neutral background
  - Fairly straightforward to model with existing fluid tools (high enough collisionality for Maxwellian assumption)

Focus on Formation → develop hierarchy of tools to understand plasmoid formation, test spacecraft / integration and design new thrusters

Objectives

- **Multi-scale / multi-physics capability –** emphasis is on high order methods and consistent collision modules
- **Formation and acceleration dynamics –** validation with experimental test campaign
- **FRC stability and turbulence onset –** fundamental physics of current sheets in plasma
- **Collisional-radiative characteristics –** detailed spectral signatures to reduce need to perturb plama

Well-developed software framework will support not only FRC simulation, but also development of future HET / plume / general partially ionized, magnetized plasma devices

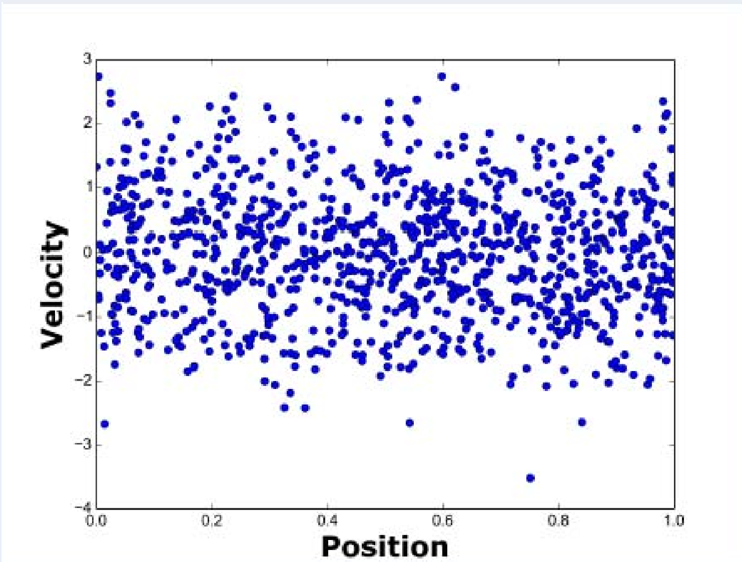
Technical Challenges

- **Field/plasma model** – Plasmas can have very complex phase space configurations and self-induced electromagnetic forces. Fundamental multiscale problem since electrostatic forces are relevant in this problem and force very small timescales.
- **Collisional Physics** – kinetic of ionization (and especially energy loss) are very sensitive to details of electron energy distribution function
- **Multi-Scale Effects** – diffusive behavior in region of separatrix is not modeled correctly in collisionless code – artificial diffusion makes computational phenomena look real.
- **System Complexity** – realistic simulations need to couple circuit models to plasma, represent three-dimensional / end effects, address radiation loss to impurities, etc.

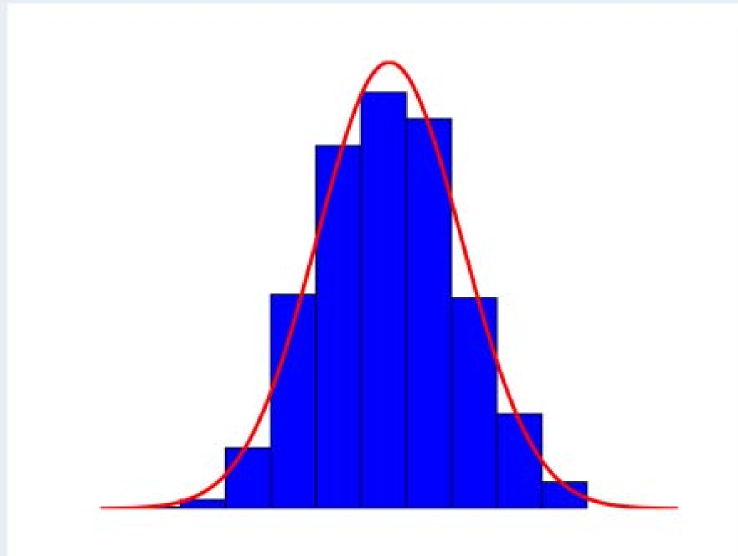
Approach

- **Hierarchy of Plasma Models**
  – Continuum: Hugrass – Ideal MHD – Hall/Resistive MHD – MF
  – Kinetic: Explicit – Implicit Particle-in-cell (PIC)
- **Collisional Physics** – Consistent derivations from underlying cross-sections for both C-R and MF coupling terms
- **Code acceleration** – Development of physics modules in next-generation simulation framework at AFRL/RQRS; leveraging both software (algorithmic) and hardware (GPU) acceleration

Plasma Description



- 3-Dimensions + 3-Velocities
- Evolve the particles position and velocity
- e.g. Particle-In-Cell models



- Ensemble average of particles distribution, $f_s(\mathbf{x},\mathbf{v},t)$
- Evolve the distribution function
- e.g. Vlasov-Maxwell models

Kinetic (non-Maxwellian)

- The Boltzmann eqn:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left. \frac{\partial f_s}{\partial t} \right|_c$$

- Take the 0^{th}, 1^{st}, 2^{nd} moments of the Boltzmann Eqn.

$$m_s \int \mathbf{v}^n \frac{\partial f_s}{\partial t} d\mathbf{v} + m_s \int \mathbf{v}^{n+1} \cdot \frac{\partial f_s}{\partial \mathbf{x}} d\mathbf{v} + q_s \int \mathbf{v}^n (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} d\mathbf{v} = m_s \int \mathbf{v}^n \left. \frac{\partial f_s}{\partial t} \right|_c d\mathbf{v}$$

- Each moment of the Boltzmann eqn gives an equation for the moment variable, and introduces the next higher moment variable
- This process can go on indefinitely

Can implement Boltzmann equation multiple ways:

- **PIC** (high dimensionality – 3D3V; high statistical noise)
- Vlasov (low dimensionality – 2D2V hero runs; smooth solution)

Leverage 6.2 program to provide codes and expertise

Moments of the distribution

- Modeling each particle velocity and position is not practical.
- Instead an average is performed to give a statistical description.
- Calculate the number of particles per unit volume having approximately the velocity $\mathbf{v}$ near the position $\mathbf{x}$ and at time t, distribution function $f(\mathbf{v}, \mathbf{x}, \mathbf{t})$

$$\rho_s = m_s \int f_s(\mathbf{v}) d\mathbf{v}$$

$$\rho_s \mathbf{u}_s = m_s \int \mathbf{v} f_s(\mathbf{v}) d\mathbf{v}$$

$$\mathbb{P}_s = \mathbf{P}_s = m_s \int \mathbf{w}\mathbf{w} f_s(\mathbf{v}) d\mathbf{v}, \quad p_s = \frac{1}{3} m_s \int w^2 f_s(\mathbf{v}) d\mathbf{v}$$

$$\mathbf{H}_s = m_s \int \mathbf{w}\mathbf{w}\mathbf{w} f_s(\mathbf{v}) d\mathbf{v}, \quad \mathbf{h}_s = \frac{1}{2} m_s \int w^2 \mathbf{w} f_s(\mathbf{v}) d\mathbf{v}$$

$$\mathbf{w} = \mathbf{v} - \mathbf{u}_s$$

No assumptions on shape of distribution function yet….still a general fluid!

Multifluid (MF) Plasma Model

Fluid Description (3 equations for each specie)

Closure through LTE

Mass $$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = S_s$$

Mom. $$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot \left(\rho_s \mathbf{u}_s \mathbf{u}_s + p_s \overleftrightarrow{\mathbf{I}}\right) = \frac{\rho_s q_s}{m_s}(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \sum_r \mathbf{R}_{rs} - \nabla \cdot \overleftrightarrow{\mathbf{\Pi}}_s$$

Energy $$\frac{\partial \varepsilon_s}{\partial t} + \nabla \cdot ((\varepsilon_s + p_s)\mathbf{u}_s) = \left(\frac{\rho_s q_s}{m_s}\mathbf{E} + \sum_r \mathbf{R}_{rs}\right) \cdot \mathbf{u}_s + \overleftrightarrow{\mathbf{\Pi}}_s : \nabla \mathbf{u}_s - \nabla \cdot \mathbf{q}_s + \sum_r Q_{rs}$$

Interspecies collisional terms

Electromagnetic Fields:

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0, \quad \nabla \cdot \mathbf{B} = 0$$

$$\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -\mu_o \sum_s \frac{q_s}{m_s}\rho_s \mathbf{u}_s, \quad \epsilon_o \nabla \cdot \mathbf{E} = \sum_s \frac{q_s}{m_s}\rho_s$$

Further Simplification

- If there are no neutral species, sum all charged species to get a single charged "fluid" equation
- Connect with electromagnetism through Faraday's law and suitable version of Ohm's Law

(Faraday's Law) $$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$

<u>MHD variants of Ohm's Law</u>

Ideal	$0 = \vec{\mathbf{E}} + \vec{\mathbf{u}} \times \vec{\mathbf{B}}$	Magnetic field frozen in flow (no diffusion)
Resistive	$\vec{\mathbf{j}} = \sigma(\vec{\mathbf{E}} + \vec{\mathbf{u}} \times \vec{\mathbf{B}})$	Magnetic field can diffuse into plasma
Hall	$\vec{\mathbf{j}} = \bar{\bar{\sigma}}\vec{\mathbf{E}}$	Incorporates Hall currents
Resistive Hall	$\vec{\mathbf{j}} = \bar{\bar{\sigma}}(\vec{\mathbf{E}} + \vec{\mathbf{u}} \times \vec{\mathbf{B}})$	Hall currents and magnetic field diffusion

- Remove fluid equations altogether and simply represent plasma as finite resistivity affecting Maxwell's equations – Hugrass RMF model

Need all models in same framework to facilitate simulation at multiply levels of fidelity → can identify role of missing physics

Detailed C-R

- **Require spectral signatures to interrogate many regions of the plasma without significantly perturbing plasma**
  – Probes don't function well in RMF environment
- **Leveraging 6.2 and 6.1 work to access detailed Ar models**
  – Based on theoretical and experimental databases from NIST, LXCAT, etc.
  – 6.2 work supports detailed model development / 6.1 work has supported intelligent model reduction for computational tractability
- **Groundwork has been laid to develop self-consistent inelastic transport coefficients from same cross-section databases**

Progress Update

Numerical Model Development

- **Tackling both sides of plasma hierarchy**
  - Low-fidelity Hugrass model
    - Quick turnaround so more useful for interfacing with experimentalists
    - Potential low-order model for time-parallel acceleration strategy
  - High-fidelity Multifluid MHD model
    - Bringing SoA multifluid capability to FRC modeling
      - High order DG formulation
    - Working on extension to three-fluid
    - Maintaining consistency with C-R rates
- **Computational machinery for high-fidelity will be used to build MHD (single fluid) system**

Hugrass model

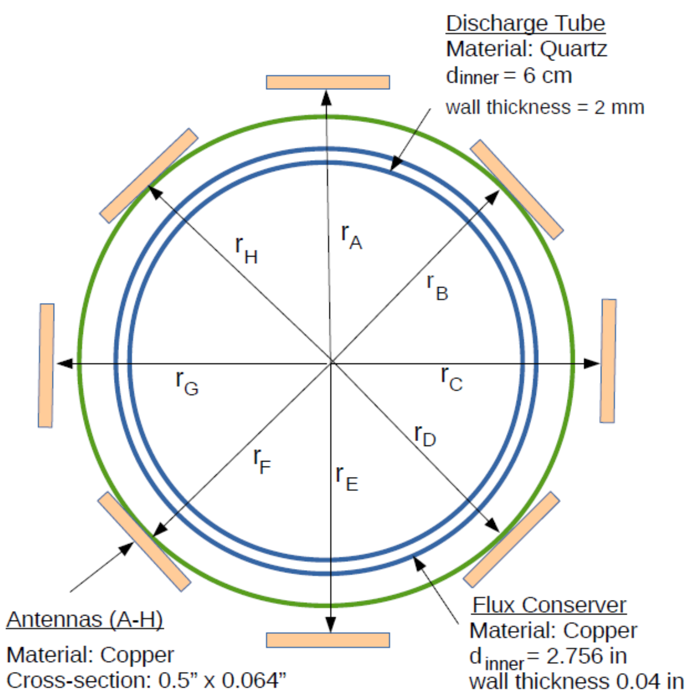
- Implementation of Maxwell's equations in non-vacuum
- Based on numerical model originally published in 1981 by Hugrass (J. Plasma Physics, **26**, 455-464)
- Radial-azimuthal code (infinitely long cylinder)
- Coupled equations for axial magnetic field and axial component of magnetic vector potential; solved spectrally

$$\frac{\partial A_z}{\partial t} = \frac{\eta}{\mu_0}\nabla^2 A_z + \frac{1}{n_e e \mu_0 r}\left[\left(\frac{\partial A_z}{\partial r}\right)\left(\frac{\partial B_z}{\partial \theta}\right) - \left(\frac{\partial A_z}{\partial \theta}\right)\left(\frac{\partial B_z}{\partial r}\right)\right]$$

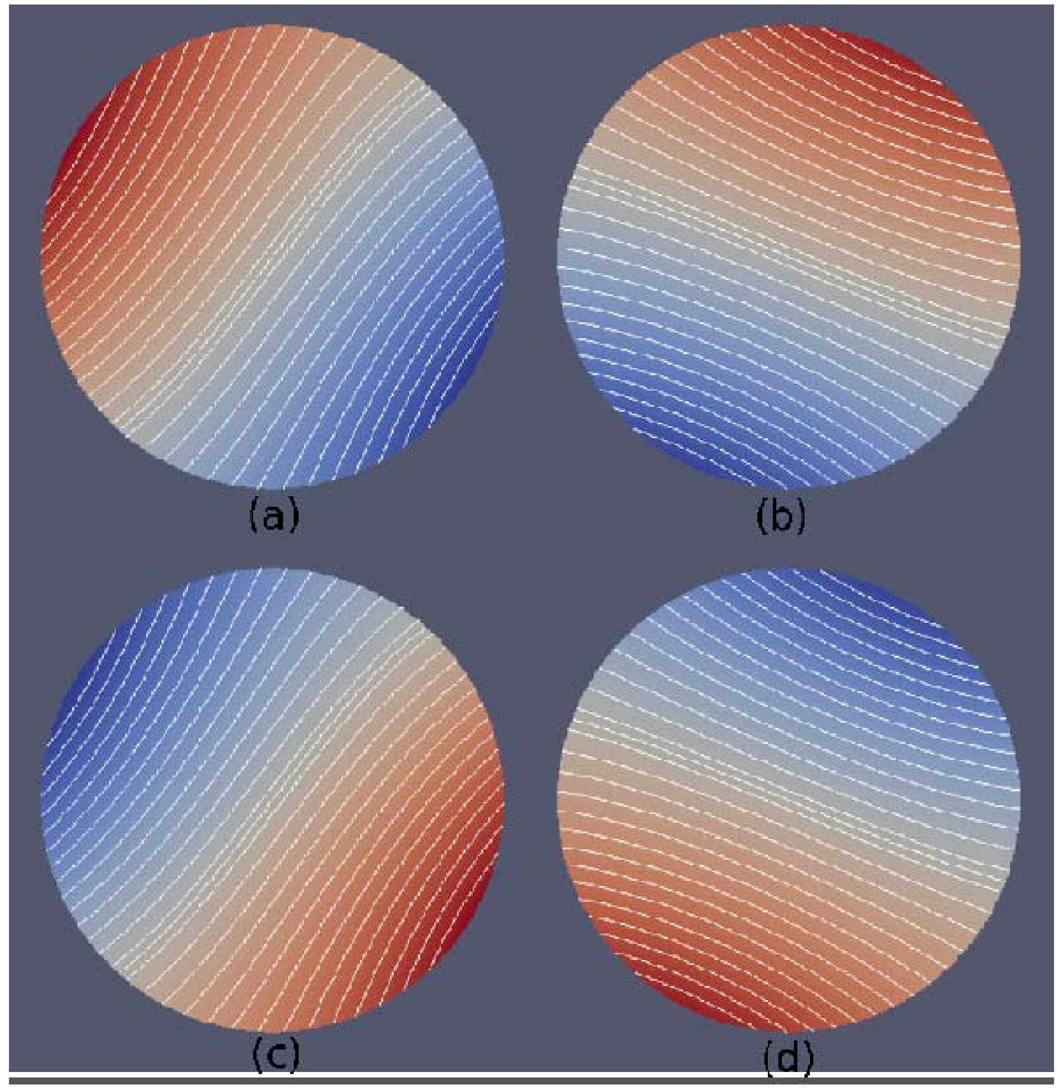
$$\frac{\partial B_z}{\partial t} = \frac{\eta}{\mu_0}\nabla^2 B_z + \frac{1}{n_e e \mu_0 r}\left[\left(\frac{\partial A_z}{\partial \theta}\right)\frac{\partial}{\partial r}\nabla^2 A_z - \left(\frac{\partial A_z}{\partial r}\right)\frac{\partial}{\partial \theta}\nabla^2 A_z\right]$$

- No self-consistent plasma transport – can still define spatial/temporal dependence of resistivity (η)

Vacuum field of RMF FRC



$$r_{A,B} = -I(t)\cos(\omega_{RMF}t + \varphi)$$
$$r_{C,D} = I(t)\sin(\omega_{RMF}t)$$
$$r_{E,F} = I(t)\cos(\omega_{RMF}t + \varphi)$$
$$r_{G,H} = -I(t)\sin(\omega_{RMF}t)$$



Development of MFPM

$$\text{Kinetic} \xrightarrow{\text{LTE, velocity moments}} \text{MFPM} \xrightarrow{\epsilon_o \to 0, \quad m_e \to 0} \text{MHD}$$

Ideal MHD model is valid when:

- High collisionality, $\tau_{ii}/\tau \ll 1$
- Small Larmor radius, $r_{Li}/L \ll 1$
- Low Resistivity, $\left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{r_{Li}}{L}\right)^2 \frac{\tau}{\tau_{ii}} \ll 1$

Multi-Fluid Plasma Model

- Less computationally expensive than kinetic models
- Multi-fluid effects become relevant at small spacial and temporal scales
- Finite electron mass and speed-of-light effects are included
- There is charge separation is modeled
- Displacement current effects are resolved in the MFPM

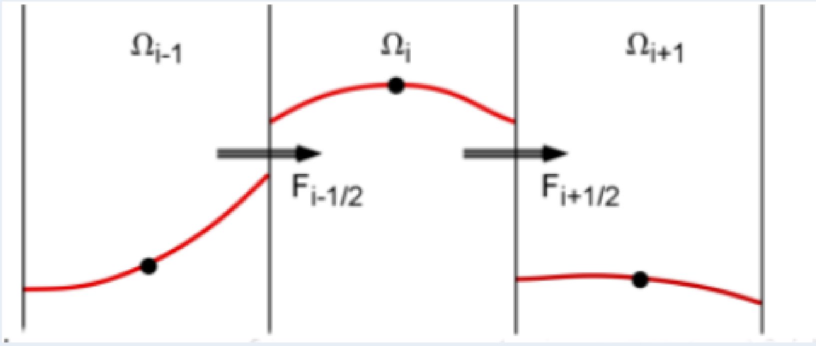
Discontinuous Galerkin (1)

- Conservation Laws are given by $\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S}$
- Multiply by basis functions and integrate over the volume

$$\int_{\Omega} v_h \frac{\partial \mathbf{Q}}{\partial t} dV + \oint_{\partial\Omega} v_h \mathbf{F} \cdot d\mathbf{A} - \int_{\Omega} \mathbf{F} \cdot \nabla v_h dV = \int_{\Omega} v_h \mathbf{S} dV$$

- That is an ODE of the form $\frac{d\mathbf{Q_h}}{dt} = \mathcal{L}_r(\mathbf{Q})$, solved using 2^{nd}, 3^{rd} or 4^{th} order TVD Runge-Kutta

Spatial order of 2nd-16th is common.



Discontinuous Galerkin (2)

- Riemann problems are solved at each interface to compute fluxes
- The source of dissipation / dispersion depends on the Riemann solver
- Variables are allowed to be discontinuous at the cell interfaces

Advantages

- Method is conservative
- Compact stencil for flux evaluation (easily parallelizable)
- Provides good flux-source coupling (no source splitting is needed)
- Resolves fast oscillations (e.g. plasma frequency) and large gradients (e.g. shocks)

Disadvantages

- Explicit Runge-Kutta Discontinuous Galerkin CFL limit is < 1/(2p-1) where high order schemes have higher p

Multiscale issues

Characteristic speeds:

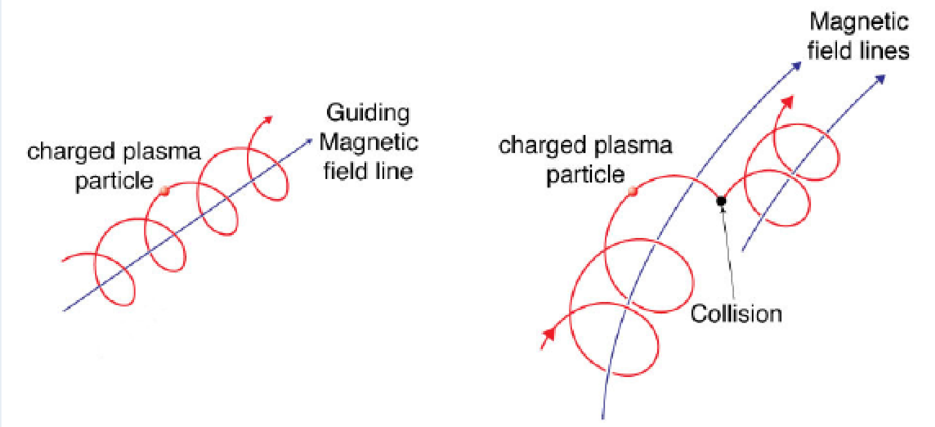
$$v_{cs} = \sqrt{\gamma \frac{P_s}{\rho_s}}, \quad c$$

Characteristic frequencies:

$$\omega_{ps} = \sqrt{\frac{n_s q_s^2}{\epsilon_o m_s}},$$

$$\omega_{cs} = \frac{q_s B}{m_s}$$

$$\nu_{sr} = \frac{4\sqrt{\pi} e^4 Z^4 n_r \log \Lambda}{3\sqrt{m_s} T_s^{3/2}}$$

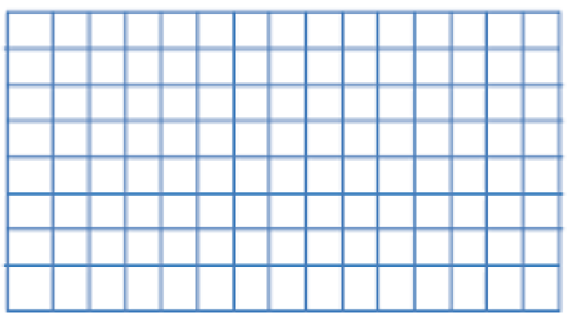


- Example lab plasma (seconds): $\frac{1}{\omega_{pe}} = 10^{-14}, \frac{L}{c} = 10^{-9}, \frac{1}{\omega_{ci}} = 10^{-8}, \frac{L}{v_{ci}} = 10^{-5}$

Availability

Structured DG:

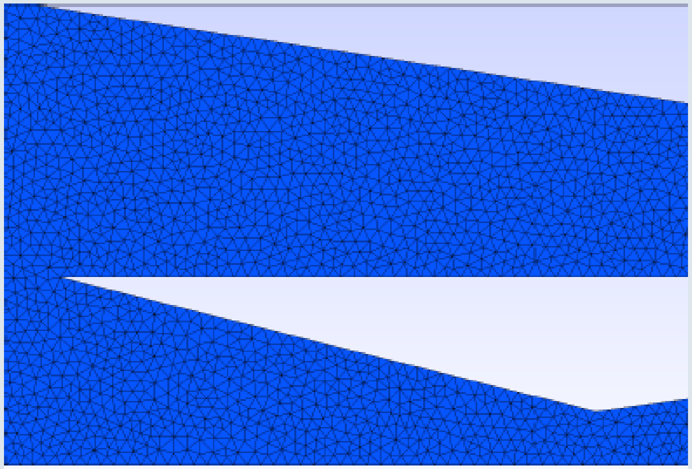
- WARPX code developed at Washington
- Validated and with proven results
- Can be used to verify the unstructured case



Structured Grid

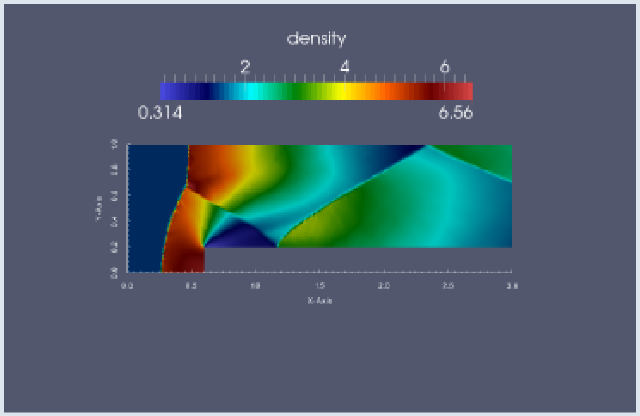
Unstructured DG:

- Currently being developed at AFRL
- For complex geometries/experiments
- Will include different physics modules

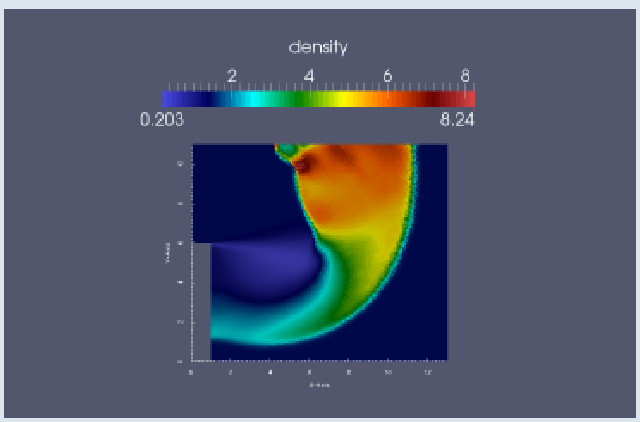


Verification

Forward Facing Step



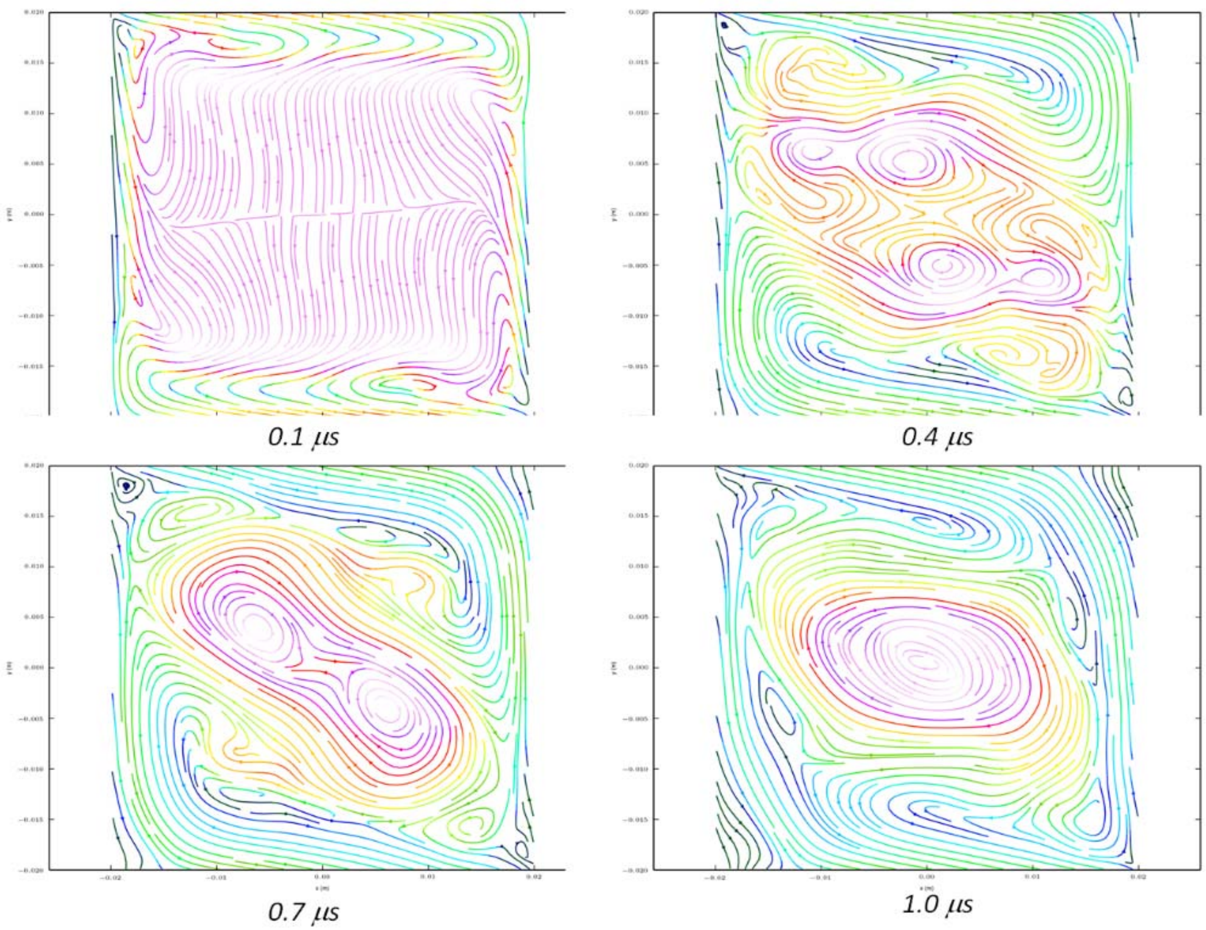
Backward Facing Step



Implementation Pieces

- Unstructured grid readers
- Unstructured data structures
- PETCs library for data manipulation and time integration
- XML input file reader
- Quadrature rules integration
- Different Riemann solvers for numerical flux (e.g. Roe, HLL, HLLE)
- Flux Limiters for shock capturing
- Initial/boundary conditions

RMF startup (1)

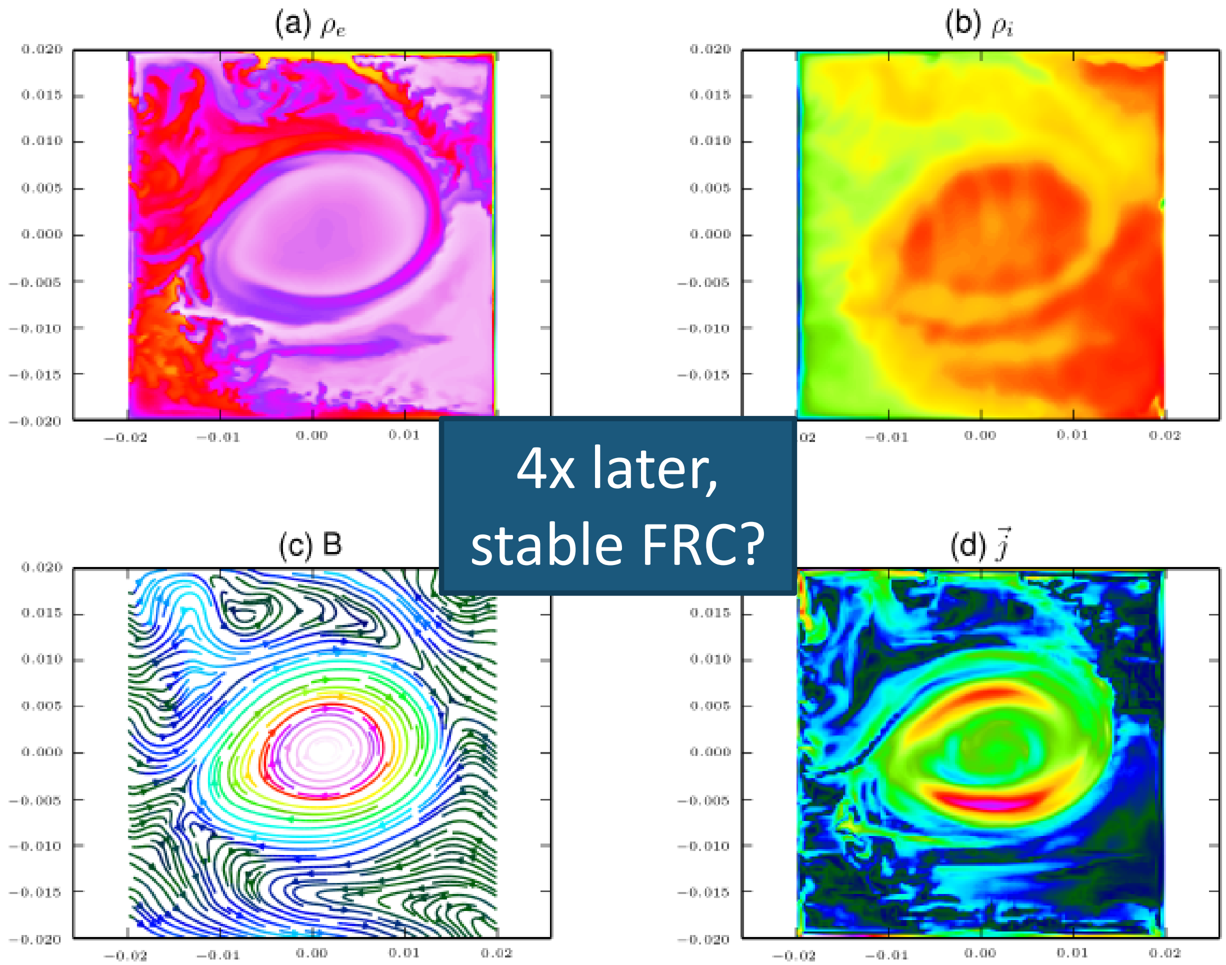


Exploring collisionless limit for efficient FRC formation

RMF startup (2)

Electron mass density

Ion mass density



Magnetic field

Current

Next Steps

- **Numerical**
  - – Develop/run continuum plasma hierarchy
  - – Run kinetic plasma codes
  - – Integrate with collisional physics
  - – Code acceleration
- **Physics**
  - – FRC formation validation
    - Magnetic reconnection
    - Instability modeling

Conclusions

Project Highlights

- **Leverages 6.1 and 6.2 efforts**
  – Explicit Particle-In-Cell (PIC) and implicit PIC development for Hall Effect Thrusters (6.2, AFRL/RQRS)
  – C-R model reduction (6.1, Luginsland/Marshall)
  – Algorithm acceleration (6.1, Farhoo)
- **Integration of plasma modules into framework code provides tools for investigation of numerous plasma phenomena**
  – Adapt Multifluid description to study electrostatic anomalous electron transport due to fluid instabilities
  – Use Multifluid / MHD description as preconditioner for implicit PIC
  – Detailed C-R necessary to evaluate utility of reduced models (such as Quasi-Steady State (QSS) approximation)

Common simulation framework has potential to greatly increase transition opportunities by decreasing integration time

Summary

- **Research plan directly supports R&D objectives for advanced plasma propulsion (i.e. FRCs)**
- **Progress made in various areas:**
  - Moving towards *consistent* plasma hierarchy
  - Integrating additional physics modules for real engineering utility
  - Need more algorithm work (accuracy/stiffness)
- **Fundamental work, can be applied to other areas**
- **Challenging, *exciting* project….**
- **….but off to a slow start**